Midterm Exam Complex Analysis
16/12/13, 09.00–11.00

1. Let $f(z)$ be a complex function, $z_0 \in \mathbb{C}$. Assume that $f(z)$ is differentiable in z_0. Let $g(z)$ be such that for all $z \neq z_0$

$$f(z) = g(z)(z - z_0) + f(z_0)$$

$\frac{x^2+y^2}{x+y}$ $\qquad$ $\frac{2x+2y}{}$

   (a) Show that $\lim_{z \to z_0} g(z) = f'(z_0)$

   (b) Show that $f(z)$ is continuous in z_0.

2. Consider the function $f(z) = |z|^2$ on $\mathbb{C}$.

$\forall \varepsilon > 0 \ \exists \delta$ $\qquad$ $0 < |f(z) - L| < \delta$

$|z - 0| < \varepsilon$

   (a) Prove that $f(z)$ is differentiable in $z = 0$.

   (b) Is $f(z)$ analytic in $z = 0$? Explain your answer.

3. Let $f(z)$ be an entire function with the property that $f(z) \in \mathbb{R}$ for all $z \in \mathbb{C}$. Prove that $f(z)$ is a constant function.

4. In this problem, we denote by $log(z)$ the logarithmic multi-valued function. By $Log(z)$ we denote the principal branch of this function.

   (a) Prove that $e^{log(z)} = z$ for all $z \neq 0$.

$e^{i \cdot -i} + e^{-i \cdot -i} - e^{i \cdot i} + e^{-i \cdot i}$

   (b) Compute $log(i)$

   (c) Determine z such that $Log(z) = \frac{1}{4}\pi i$.

$e + e^{-1} - e^{-1} - e$

5. Let Γ be any contour with initial point $z = i$ and terminal point $z = -i$. Compute

$$\int_\Gamma \sin(z)dz$$

6. Consider the complex function $f(z) = \frac{1}{z^2-1}$.

   (a) Compute a partial fraction decomposition of $f(z)$.

   (b) Late Γ_1 be the positively oriented closed contour $|z| = 2$. Compute the integral $\int_{\Gamma_1} f(z)dz$.

Points: 1: 7 + 8, 2: 8 + 7, 3: 15, 4: 5 + 5 + 5 , 5: 15 , 6: 5 + 10; 10 for free.